MAT 2377 (Winter 2014)

Solution to Assignment 1

[2]

3.26. The random variable X has a binomial distribution with $n = 4$ and $p = 0.0001$. We have

$$P(X = k) = \binom{4}{k}(0.0001)^k(1 - 0.0001)^{4-k}, k = 0, 1, 2, 3, 4.$$

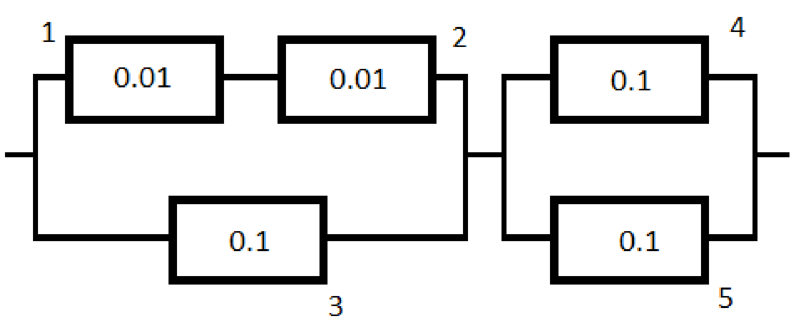
[2]

3.46. We have $F(x) = P(X \le x)$. Therefore

$$F(x) = \begin{cases} 0 & \text{if } \ x < 0, \\ 0.9996001 & \text{if } \ 0 \le x < 1, \\ 0.9996001 + 0.00039988 & \text{if } \ 1 \le x < 2, \\ 0.9996001 + 0.00039988 + 5.9988(10^{-8}) & \text{if } \ 2 \le x < 3, \\ 0.9996001 + 0.00039988 + 5.9988(10^{-8}) + 3.9988(10^{-12}) & \text{if } \ 3 \le x < 4, \\ 1 & \text{if } \ x \ge 4. \end{cases}$$

[5]

2-208:



Define

$$A_i = \text{The } \ i^{th} \text{ device operates}, i = 1, 2, 3, 4, 5.$$

The device operates if $((A_1 \cap A_2) \cup A_3) \cap (A_4 \cup A_5)$ occurs. Since the devices work independently, we have

$$P(((A_1 \cap A_2) \cup A_3) \cap (A_4 \cup A_5)) = P((A_1 \cap A_2) \cup A_3)P(A_4 \cup A_5).$$

Since

$$P(A_4 \cup A_5) = P(A_4) + P(A_5) - P(A_4)P(A_5) = 0.9 + 0.9 - (0.9)(0.9) = 0.99$$

and

$$P((A_1 \cap A_2) \cup A_3) = P(A_1 \cap A_2) + P(A_3) - P(A_3 \cap (A_1 \cap A_2))$$

$$= 0.9801 + 0.9 - (0.9)(0.99)(0.99) = 0.99801.$$

Therefore

$$P(((A_1 \cap A_2) \cup A_3) \cap (A_4 \cup A_5)) = 0.9880299.$$

[5]

3-52:

(a):

$$P(X \leq 1/18) = F(1/18) = 0$$

(b):

$$P(X \leq 1/4) = F(1/4) = 0.9.$$

(c)

$$P(X \leq 5/16) = F(5/16) = 0.9$$

(d)

$$P(X > 1/4) = 1 - P(X \leq 1/4) = 1 - 0.9 = 0.1.$$

(e)

$$P(X \leq 1/2) = F(1/2) = 1.$$

[4]

3-65: We have

$$E(X) = 0P(X = 0) + 1P(X = 1) + 2P(X = 2) + 3P(X = 3) + xP(X = x) = 6.$$

Since

$$P(X = 0) = P(X = 1) = P(X = 2) = P(X = 3) = P(X = x)$$

and

$$\sum_k P(X = k) = 1$$

we have

$$P(X = 0) = P(X = 1) = P(X = 2) = P(X = 3) = P(X = x) = \frac{1}{5}.$$

Therefore

$$6 = 1/5 + 2/5 + 3/5 + x/5.$$

Solve for x to get $x = 24$.

[1]

3-110: **Remark:** It is 1 mark for part (a). A student that has successfully answered part (b) will get one bonus mark. A student that has successfully answered part (c) will get one bonus mark.

(a) Define X = number of defective components in 100 orders. Then

$$P(X = 0) = 0.98^{100} = 0.1326196.$$

(b) Define X = the number of defective components in 102 orders and let Y = the number of defective component in 100 orders coming from the first order. We need to calculate $P(Y = 0)$. Using total probability rule

$$P(Y = 0) = P(Y = 0|X = 0)P(X = 0)+P(Y = 0|X = 1)P(X = 1)+P(Y = 0|X = 2)P(X = 2).$$

Notice that

$$P(Y = 0|X = 0) = 1, P(Y = 0|X = 1) = \frac{\binom{101}{100}}{\binom{102}{100}} = \frac{2}{102}, P(Y = 0|X = 2) = \frac{\binom{100}{100}}{\binom{102}{100}} = \frac{2}{(101)(102)}.$$

On the other hand

$$P(X = 0) = 0.98^{102}, P(X = 1) = \binom{102}{1}(0.02)^1(0.98)^{101}, P(X = 2) = \binom{102}{2}(0.02)^2(0.98)^{100}.$$

This is equivalent to

$$P(X = 0) = 0.98^{102}, P(X = 1) = 102(0.02)^1(0.98)^{101},$$

$$P(X = 2) = \frac{(102)(101)}{2}(0.02)^2(0.98)^{100}.$$

Therefore using both identities we get

$$P(Y = 0) = 0.98^{102} + 2(0.02)^1(0.98)^{101} + (0.02)^2(0.98)^{100} = 0.1326196.$$

(c) Define X = the number of defective components in 105 orders and let Y = the number of defective components in 100 orders coming from the first order and let Y = the number of defective components in 100 orders coming from the first order. Similarly we get

$$P(Y = 0) = P(Y = 0|X = 0)P(X = 0) + P(Y = 0|X = 1)P(X = 1)$$

$$+P(Y = 0|X = 2)P(X = 2) + P(Y = 0|X = 3)P(X = 3)$$

$$+P(Y = 0|X = 4)P(X = 4) + P(Y = 0|X = 5)P(X = 5).$$

Similar to the previous part $P(Y = 0|X = 0) = 1$ and for $k = 1, 2, 3, 4, 5$ we have

$$P(Y = 0|X = k) = \frac{\binom{105-k}{100}}{\binom{105}{100}}.$$

Also

$$P(X = k) = \binom{105}{k}(0.02)^k(0.98)^{105-k}. \tag{1}$$

This gives

$$\begin{aligned}
P(Y = 0|X = 0) &= 1, \\
P(Y = 0|X = 1) &= \frac{5}{105}, \\
P(Y = 0|X = 2) &= \frac{5 \times 4}{105 \times 104}, \\
P(Y = 0|X = 3) &= \frac{5 \times 4 \times 3}{105 \times 104 \times 103}, \\
P(Y = 0|X = 4) &= \frac{5 \times 4 \times 3 \times 2}{105 \times 104 \times 103 \times 102}, \\
P(Y = 0|X = 4) &= \frac{5 \times 4 \times 3 \times 2 \times 1}{105 \times 104 \times 103 \times 102 \times 101}.
\end{aligned}$$

Using (1) and the above conditional probabilities we get exactly the same answer

$$P(Y = 0) = 0.1326196.$$

[2]

3-131: Let X be the number of transactions until the third computer fails. Then X has a negative binomial distribution with $r = 3$ and $p = 10^{-8}$. Therefore from

$$E(X) = r/p = 3/10^{-8} = 3(10^8)$$

and

$$Var(X) = rq/p^2 = 3(1 - 10^{-8})/10^{-16} = 3(10^{16}).$$

[2]

3-138:

(a) Let X be the number of defective bulbs in an automotive light. We have $X \sim Binomial(30, 0.001)$. Therefore

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.999^{30} - 30(0.001)(0.999)^{29} = 0.0004269616.$$

(b) Let T be the number of automotive lights to check to obtain a light with at least two defective bulbs. T has a geometric distribution with $p = 0.0004269616$. We want

$$E(T) = 1/p = 1/0.0004269616 = 2342.131.$$

Remark: The final numerical answer to (b) might vary slightly depending on the rounding of the answer in part (a).

[/??]